PROJECT#2 **Coefficients of Hermite polynomials by recursion relation Eq.(4-16)**

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1. Purpose

By using the Eq.(4-16), we try to get Hermite polynomials.

2. Introduction

$$\begin{cases} a_{n+1,k} = 2a_{n,k-1} - 2na_{n-1,k} & \text{where } k>0 \\ a_{n+1,k} = -2na_{n-1,k} & \text{where } k=0 \end{cases} \qquad \text{Eq.(4-16)}$$

By defining coefficients $a_{0,0} = 1$, $a_{1,0} = 0$, and $a_{1,1} = 2$ and using the Eq.(4-16), we get Hermite polynomials.

Suppose Hermite polynomials are expressed as $H_n(\rho) = \sum_{k=0,1}^{n} a_{n,k}\rho^k$.

3. Result

$$\begin{bmatrix} H_0(\rho) \\ H_1(\rho) \\ H_2(\rho) \\ H_3(\rho) \\ H_4(\rho) \\ H_5(\rho) \\ H_6(\rho) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & -12 & 0 & 8 & 0 & 0 & 0 \\ 12 & 0 & -48 & 0 & 16 & 0 & 0 \\ 0 & 120 & 0 & -160 & 0 & 32 & 0 \\ -120 & 0 & 720 & 0 & -480 & 0 & 64 \end{bmatrix} \begin{bmatrix} 1 \\ \rho \\ \rho^2 \\ \rho^3 \\ \rho^4 \\ \rho^5 \\ \rho^6 \end{bmatrix}$$

These Hn(ρ) agree with ones on the text book for n=0 to 5..

We cannot get $|a_{n,k}|$ over 16 decimal digits because of the double precision..

Fig.1 shows Hn(ρ)/n^3 for n=1 to 6.

By plugging Hn(ρ) into the equation below, we get one-dimensional harmonic oscillator wave functions.

$$\Psi_n(\rho) = \frac{1}{\sqrt{2^n n\sqrt{\pi}}} e^{-\frac{\rho^2}{2}} H_n(\rho)$$

Fig.2 shows $\Psi_n(\rho)$ for n=0 to 3 and agrees with Fig. 4-2 on the textbook.

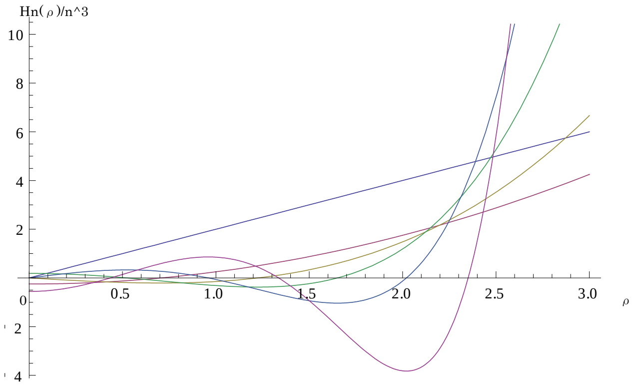


Fig.1 Hermite polynomials divided by n^3 for n=0 to 6.

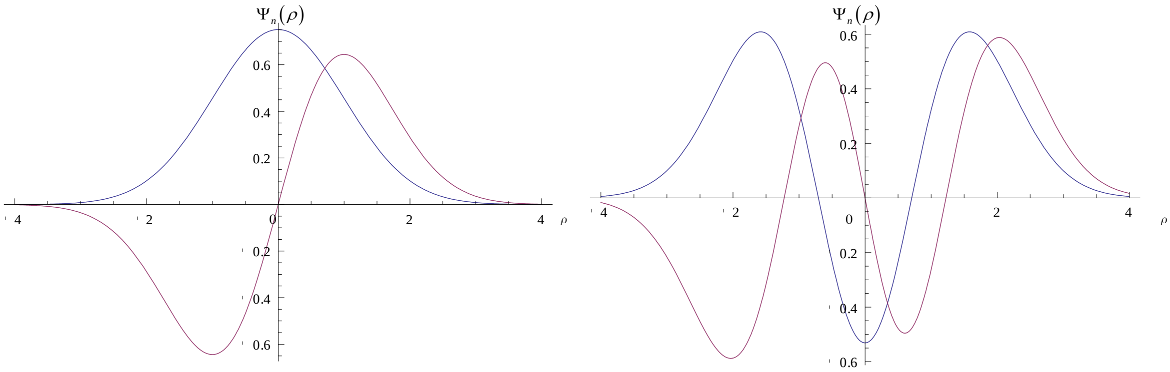
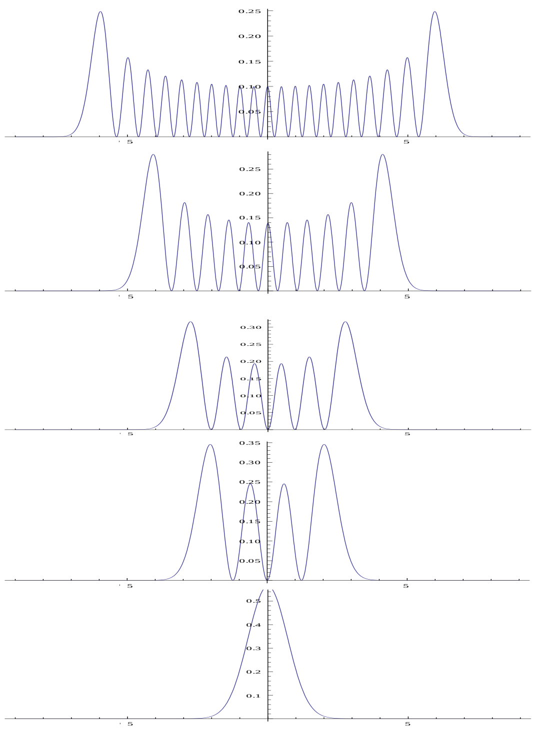


Fig.2 One-dimensional harmonic oscillator wave functions Ψn(ρ) for n=0 to 3.



As n becomes large, particle behaves as classic pendulum because it spends most of time at the ends.

Fig.3 Pribability |Ψn(ρ)|^2 for n=1,3,5,10, and 20.

4.Reference

[1] The textbook

Samuel SM Wong. *Computational Methods in Physics and Engineering 2nd edition.* World Scientific Publishin Co. Pte. Ltd, 1997.